

Mutually Exclusive and Overlapping Events

GOAL Find the probability of mutually exclusive, overlapping, and complementary events.

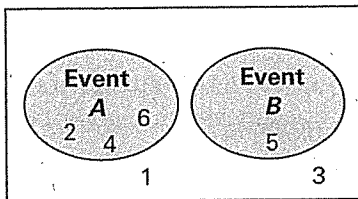
When you consider the probability of two events occurring, the events are called **compound events**. *Mutually exclusive* and *overlapping* events are examples of compound events.

Mutually exclusive events are events that have no outcomes in common.

Overlapping events are events that have one or more outcomes in common. The Venn diagrams below show how the events that involve rolling a number cube are related.

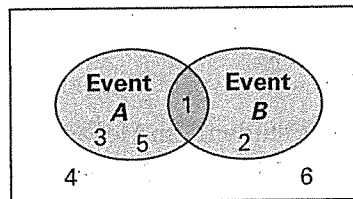
Mutually Exclusive Events

Event A: Get an even number.
Event B: Get a 5.



Overlapping Events

Event A: Get an odd number.
Event B: Get a number less than 3.



Use the formula below to find the probability of mutually exclusive events.

Probability of Mutually Exclusive Events

For two mutually exclusive events A and B , the probability that either of the events occurs is the sum of the probabilities of the events.

$$P(A \text{ or } B) = P(A) + P(B) \quad \text{Events } A \text{ and } B \text{ are mutually exclusive.}$$

EXAMPLE 1 Probability of Mutually Exclusive Events

The table below lists the number of each type of fish in a fish tank. What is the probability that a randomly chosen fish is a minnow or a rainbowfish?

Type	angelfish	goldfish	minnow	rainbowfish
Number	12	9	5	4

SOLUTION

Find the probability of each event. Then add the probabilities.

$$P(\text{minnow}) = \frac{5}{30} \quad \text{Of the 30 fish, 5 are minnows.}$$

$$P(\text{rainbowfish}) = \frac{4}{30} \quad \text{Of the 30 fish, 4 are rainbowfish.}$$

$$P(\text{minnow or rainbowfish}) = \frac{5}{30} + \frac{4}{30} = \frac{9}{30} = 0.3$$

The probability that the fish is a minnow or a rainbowfish is 30%.

Probability of Overlapping Events

For two overlapping events A and B , the probability that either of the events occurs is the sum of the probabilities of the events, minus the probability that both events occur.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad \text{Events } A \text{ and } B \text{ are overlapping.}$$

EXAMPLE 2 Probability of Overlapping Events

A high school has four soccer teams. The number of players on each team is shown in the table. A soccer player is randomly chosen. What is the probability that the player is a female student or a player on a varsity soccer team?

	Girls	Boys
Junior Varsity	19	17
Varsity	23	21

SOLUTION

Let event A be "choose a female student" and let event B be "choose a player on a varsity soccer team." The events are overlapping, so find the probabilities of event A , event B , and events A and B .

$$P(A) = \frac{19 + 23}{19 + 23 + 17 + 21} = \frac{42}{80} \quad \text{Of the 80 players, 42 are female.}$$

$$P(B) = \frac{23 + 21}{19 + 23 + 17 + 21} = \frac{44}{80} \quad \text{Of the 80 players, 44 are on varsity.}$$

$$P(A \text{ and } B) = \frac{23}{80} \quad \text{Of the 80 players, 23 are on girls' varsity.}$$

Then find $P(A \text{ or } B)$.

$$P(A \text{ or } B) = \frac{42}{80} + \frac{44}{80} - \frac{23}{80} = \frac{63}{80} \approx 0.788$$

The probability that the player is female or on varsity is about 78.8%.

CHECK Examples 1 and 2

Tell whether the situation describes *mutually exclusive* or *overlapping events*. Then answer the question.

1. A gumball machine contains 25 red, 18 orange, and 17 yellow gumballs. What is the probability that a randomly chosen gumball is orange or yellow?
2. Each whole number from 1 through 10 is written on a separate piece of paper and placed in a bag. What is the probability that a randomly chosen number is an even number or a number greater than 5?

Complementary Events Two mutually exclusive events in which one or the other event *must* occur are called **complementary events**.

Probability of Complementary Events

The sum of the probabilities of two complementary events is 1. If A and B are complementary events, then $P(A) + P(B) = 1$.

The above formula means that for an event A and its complement, which may be written as “not A ,” $P(A) + P(\text{not } A) = 1$ and $P(\text{not } A) = 1 - P(A)$.

EXAMPLE 3 Using Complementary Events

There is an 80% chance that it will rain tomorrow. What is the probability that it will not rain tomorrow?

SOLUTION

The event that it rains tomorrow and the event that it does not rain tomorrow are complementary events.

$$\begin{aligned} P(\text{not rain}) &= 1 - P(\text{rain}) \\ &= 1 - 0.8 = 0.2 \end{aligned}$$

The probability that it will not rain tomorrow is 20%.

CHECK Example 3

3. The probability that your raffle ticket is not a winner is 0.9998. What is the probability that your raffle ticket is a winner?

EXERCISES

Tell whether the events are *mutually exclusive* or *overlapping*.

- You randomly choose a letter from the word MISSISSIPPI.
Event A: You get an S.
Event B: You get an I.
- You conduct a survey about pet ownership.
Event A: The respondent is a cat owner.
Event B: The respondent is a dog owner.
- You randomly choose a student in your class.
Event A: The student plays baseball.
Event B: The student plays basketball.
- You randomly choose a member of your school band.
Event A: The student plays the clarinet.
Event B: The student does not play the clarinet.

Events A and B are mutually exclusive. Find the missing probability.

5. $P(A) = 0.62$

$P(B) = 0.1$

$P(A \text{ or } B) = \underline{\quad?}$

6. $P(A) = 0.5$

$P(B) = \underline{\quad?}$

$P(A \text{ or } B) = 0.9$

7. $P(A) = \underline{\quad?}$

$P(B) = 12.5\%$

$P(A \text{ or } B) = 80\%$

Events A and B are overlapping. Find the missing probability.

8. $P(A) = 0.375$

$P(B) = 0.5$

$P(A \text{ and } B) = 0.3$

$P(A \text{ or } B) = \underline{\quad?}$

9. $P(A) = 0.64$

$P(B) = 0.24$

$P(A \text{ and } B) = \underline{\quad?}$

$P(A \text{ or } B) = 0.6$

10. $P(A) = \underline{\quad?}$

$P(B) = 16\%$

$P(A \text{ and } B) = 10\%$

$P(A \text{ or } B) = 25\%$

Tell whether the situation describes *mutually exclusive* or *overlapping events*. Then answer the question.

- You roll a number cube. What is the probability that you roll an even number or a number less than 5?
- You roll a number cube. What is the probability that you roll an odd number or a 6?
- An aquarium has 38 African penguins (21 males and 17 females) and 15 rockhopper penguins (6 males and 9 females). What is the probability that a randomly chosen penguin is a male or a rockhopper penguin?
- Of the pre-owned vehicles at a dealership, 9 are one year old, 7 are two years old, 4 are three years old, and the remaining 10 are four years old or older. What is the probability that a randomly chosen pre-owned vehicle is one or two years old?

In Exercises 15–17, use the following information.

A bag contains red, blue, and yellow marbles. The probability of choosing a red marble is 0.35. The probability of choosing a blue marble is 0.2.

- What is the probability of not choosing a red marble?
- What is the probability of not choosing a blue marble?
- What is the probability of choosing a yellow marble?

In Exercises 18–20, use the following information.

The word *or* is represented in set notation by the union (\cup) of sets. This is because an outcome is in event A or event B if and only if the outcome is in the union of the sets, $A \cup B$.

- What is $P(A \cup C)$ for the events at the right?
- What is $P(B \cup C)$ for the events at the right?
- What is $P(A \cup B)$ for the events at the right? Explain why your result makes sense.

